M. Sc. Sem III

11th Paper

Functional Analysis

Unit 1

Normed linear space : It is a linear space N in which to each vector x, there corresponds a real number, denoted by $\|x\|$ and called the norm of x in such a manner that

(a) $\|x\| \geq 0$, and $\|x\| = 0 \Leftrightarrow x = 0$

(b) $\|x+y\| \leq \|x\| + \|y\|$

(c) $\|\alpha x\| = |\alpha| \|x\|$

The non-negative real number $\|x\|$ is considered as the length of the vector x. If we consider $\|x\|$ as a real function defined on N, this function is called the norm on N.

* The normed linear space N is a metric space with respect to the metric d defined by

$$d(x,y) = \|x-y\|.$$

Q(A) Prove that

$$\|x\| - \|y\| \leq \|x-y\|$$

Proof

$$\|x\| = \|(x-y)+y\|$$

$$\Rightarrow \|x\| \leq \|x-y\| + \|y\|$$

$$\Rightarrow \|x\| - \|y\| \leq \|x-y\| \quad \text{—— (1)}$$

Q(B) Prove that

$$\big|\|x\| - \|y\|\big| \leq \|x-y\|$$

Soln Consider $-(\|x\| - \|y\|)$.

$$-(\|x\| - \|y\|) = \|y\| - \|x\|$$

$$\Rightarrow -(\|x\| - \|y\|) \leq \|y-x\| \quad [\text{Proved earlier}]$$

$$\Rightarrow -(\|x\| - \|y\|) \leq \|-(x-y)\| \text{ as } \|y-x\| = \|-(x-y)\|$$

$$\Rightarrow -(\|x\| - \|y\|) \leq \|x-y\| \quad \text{—— (2)}$$

combining (1) and (2), we have

$$\big|\|x\| - \|y\|\big| \leq \|x-y\|.$$